

# Multi-Fidelity Surrogate Model Approach to Optimization

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## ABSTRACT

Recently the use of Radial Basis Functions (RBF) has been introduced as an optional alternative to co-Kriging in the context of multi-fidelity surrogate modeling. In this paper, we compare the performance of Random Forest-based *co-surrogates* to the previously introduced co-Kriging and co-RBF using a set of bi-fidelity benchmark problems in 2, 4 and 8 dimensions. Our results show that there is a minimal overall difference between the different co-surrogate models with regards to final performance, although the training of Random Forests takes much less time compared to the Kriging and RBF methods.

## CCS CONCEPTS

• **Theory of computation** → **Random search heuristics**; • **Computing methodologies** → **Machine learning approaches**;

## KEYWORDS

Evolution Strategies, Empirical study, Surrogate Modeling

## ACM Reference Format:

Sander van Rijn ✉, Sebastian Schmitt, Markus Olhofer, Matthijs van Leeuwen, and Thomas Bäck. 2018. Multi-Fidelity Surrogate Model Approach to Optimization. In *GECCO '18 Companion: Genetic and Evolutionary Computation Conference Companion, July 15–19, 2018, Kyoto, Japan*. ACM, New York, NY, USA, Article 4, 2 pages. <https://doi.org/10.1145/3205651.3205757>

## 1 INTRODUCTION

Optimization problems in engineering domains often rely on simulations to determine the quality of candidate solutions. Numerical optimization methods such as CMA-ES [5] allow automation of such processes, but require many hundreds or thousands of evaluations. As the required amount of runtime per simulation increases, more advanced optimization methods are needed that can work with fewer simulations.

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GECCO '18 Companion, July 15–19, 2018, Kyoto, Japan

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ACM ISBN 978-1-4503-5764-7/18/07...\$15.00

<https://doi.org/10.1145/3205651.3205757>

To this end, Surrogate-Assisted optimization methods such as [3, 7] were developed that use surrogate models (also called meta-models) as a less computationally expensive substitute for the simulations. Co-Kriging was created by combining surrogate modeling with multi-fidelity simulations by [4], assuming that a lower fidelity simulation contains some global fitness landscape information.

In this paper we introduce co-Random Forests to the set of co-surrogates consisting only of co-Kriging and co-RBF [2]. We compare the performance of these three co-surrogates on eight benchmark functions as used in co-Surrogate-Assisted CMA-ES (cSA-CMA-ES) optimization.

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## Algorithm 1 cSA-CMA-ES

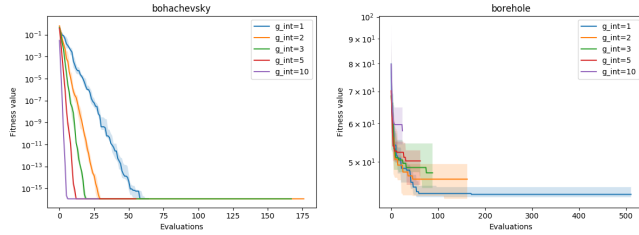
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1:  $t, \lambda_{\text{pre}}, \vec{x}, \vec{x}_{\text{best}}, y_{\text{best}} \leftarrow 0, 2\lambda$ , random individual, Null,  $\infty$ 
2: DoE  $\leftarrow$  Design of Experiments sample
3:  $\mathbb{A} \leftarrow (\text{DoE}, f_h(\text{DoE}), f_l(\text{DoE}))$ 
4: while not terminate do
5:    $\rho \leftarrow$  coefficient of linear regression on  $\mathbb{A}_h, \mathbb{A}_l$ 
6:    $S_{\text{co}} \leftarrow$  train surrogate on  $\mathbb{A}_h - \rho \mathbb{A}_l$ 
7:    $O_{\text{pre}}^t \leftarrow$  mutate( $\vec{x}, \lambda_{\text{pre}}$ )
8:    $\vec{y}_l \leftarrow f_l(O_{\text{pre}}^t)$ 
9:    $\vec{y}_h \leftarrow \rho \vec{y}_l + S_{\text{co}}.\text{predict}(O_{\text{pre}}^t)$ 
10:   $O^t \leftarrow$  preselect( $O_{\text{pre}}^t, \vec{y}_h, \lambda$ )
11:  if  $t \bmod g_{\text{int}} = 0$  then
12:     $\vec{y} \leftarrow f_h(O^t)$ 
13:     $\mathbb{A} \leftarrow \mathbb{A} \cup (O^t, \vec{y}_h, \vec{y}_l)$ 
14:    if  $\min(\vec{y}) < y_{\text{best}}$  then
15:       $y_{\text{best}}, \vec{x}_{\text{best}} \leftarrow \min(\vec{y}), O_{\text{argmin}}^t$ 
16:    end if
17:  else
18:     $\vec{y}_h \leftarrow \hat{y}_h$ 
19:  end if
20:   $P^t \leftarrow$  select( $O^t, \vec{y}_h, \mu$ )
21:   $\vec{x} \leftarrow$  recombine( $P^t$ )
22:  updateInternalParameters();  $t \leftarrow t + 1$ 
23: end while
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## 2 CO-SURROGATE ASSISTED CMA-ES

Co-surrogates are based on the autoregressive model by [6] that describes a high fidelity prediction as  $\hat{f}_h(\vec{x}) = \rho f_l(\vec{x}) + \delta(\vec{x})$ , where  $f_h$  and  $f_l$  are the high and low fidelity evaluation functions,  $\hat{f}_h$



**Figure 1: Comparison of generational evolution control intervals  $g\_int$  in cSA-CMA-ES using Random Forests for two example benchmark functions (2d and 8d).**

is the predicted value for  $f_h$  based on the co-surrogate model,  $\rho$  is a scaling parameter and  $\delta(\vec{x})$  is an error term for the difference between the high and low fidelity results. Co-surrogates are used to predict this  $\delta$ , which is calculated as  $\delta(\vec{x}) = f_h(\vec{x}) - \rho f_l(\vec{x})$  to create training data for the co-surrogate.

Algorithm 1 shows the cSA-CMA-ES that we use in this paper. We compare it with a similar SA-CMA-ES, that mainly differs in lines 5–9 where the low fidelity function  $f_l$  is used for the high fidelity surrogate predictions. They make use of both *pre-selection* (line 10) and *generational evolution control* (lines 11–19).

### 3 EXPERIMENTS

For all experiments, an initial Latin-Hypercube-Sample of 20 points was evaluated as DoE to be used as initial training set for the surrogates. For pre-selection,  $\lambda_{pre}$  is fixed to  $2\lambda$ . Each run has been repeated ten times. For RBF, we use the multiquadric basis function, and for Random Forests we create 100 trees. The used benchmark functions were taken from [1, 8].

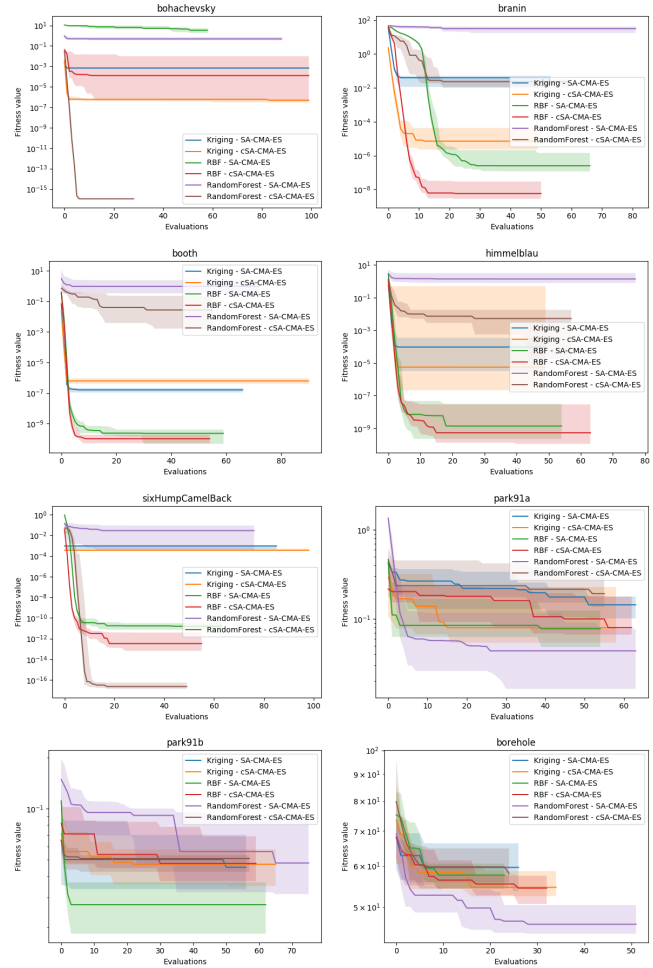
Figure 1 shows an example of the influence of generational control intervals, while Figure 2 shows the comparison between Kriging, RBF and Random Forest surrogate models for both SA-CMA-ES and cSA-CMA-ES, with a generational interval fixed to 10. In these plots, the solid lines indicates the 50<sup>th</sup> percentile of the fitness values over 10 runs, while the shaded regions indicate the spread of the fitness values between the 25<sup>th</sup> and 75<sup>th</sup> percentile. Only high fidelity evaluations are counted.

### 4 CONCLUSIONS AND OUTLOOK

In this paper, we have compared (co-)surrogate-assisted optimization performance on a collection of analytical multi-fidelity benchmark functions from literature, with surrogates based on Kriging, RBF and Random Forests. Although Kriging is generally considered to be the most stable choice for a surrogate model, our results show that the performance of the various co-surrogates is generally similar, while any significant differences dependent on the target problem specifically, as is already known in the surrogate-assisted optimization literature.

### ACKNOWLEDGMENTS

This work is part of the research program DAMIOSO with project number 628.006.002, which is partly financed by the Netherlands Organisation for Scientific Research (NWO).



**Figure 2: Comparison of Kriging, RBF and Random Forest based cSA-CMA-ES for all benchmark functions.**

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